Galina Setlak, Mikhail Alexandrov,
Krassimir Markov
(editors)

Artificial Intelligence Methods and Techniques for Business and Engineering Applications

ITHEA®
Rzeszów - Sofia
2012

Galina Setlak, Mikhail Alexandrov, Krassimir Markov (Eds.)
Artificial Intelligence Methods and Techniques for Business and Engineering Applications
ITHEA®
2012, Rzeszow, Poland; Sofia, Bulgaria,
ISBN: 978-83-63430-00-9 (PL)
ISBN: 978-954-16-0057-3 (printed)
ISBN: 978-954-16-0058-0 (online)
ITHEA IBS ISC No.: 26
First edition

Printed in Poland

Recommended for publication by The Scientific Concil of the Institute of Information Theories and Applications FOI ITHEA

This issue contains a monograph that concern actual problems of research and application of information technologies, especially the new approaches, models, algorithms and methods for information modeling to be used in business and engineering applications of intelligent and information systems.

It is represented that book articles will be interesting for experts in the field of information technologies as well as for practical users.

ISBN: 978-83-63430-00-9 (PL)
ISBN: 978-954-16-0057-3 (printed)
ISBN: 978-954-16-0058-0 (online)

C\O JUSAUTOR, SOFIA, 2012

UTILITY FUNCTION DESIGN ON THE BASE OF THE PAIRED COMPARISON MATRIX*

Stanislav Mikoni

Abstract: In the multi-attribute utility theory the utility functions are usually constructed by dots. It concerns both the lottery's method and the value increasing method. In the both cases the utility function is constructed in the absolute scale [0, 1] that causes inconveniences for experts. The comparative assessments look more preferable for decision-makers. The paired comparison matrix (PCM) looks as a natural model representing the preference structure of decision-maker (DM).

We use scale points of attributes as a PCM comparative entities. We use also increasing/decreasing entity priority as a criterion. Function of priorities is transformed to utility function on the base of a normalizing function. Such a function allows using the matrix power as parameter affecting the form of utility function.

The PCM provides the extended possibilities to DMs to form comparative assessments both the qualitative ones (as better-worse) and the quantitative ones reflecting winnings and losses of DMs. In the paper we consider methods for utility function construction having different forms of its presentation. Among them there are utility functions based on attributes measured in nominal scales.

Keywords: utility function, paired comparison matrix, scale points, priority function.

ACM Classification Keywords: A.0 General Literature - Conference proceedings H.4.2 [Information Systems Applications]: Types of Systems---decision support

Conference topic: Soft computing techniques

Introduction

The multi attribute utility theory demands a utility functions construction for each attribute. Two well known methods are used for this goal. First of them is lottery method proposed by von Neumann and Morgenstern. The second of them is based on value estimation by expert for some scale points. Both of the methods have such disadvantage as necessity of absolute quantity values assessment. According T. Saaty, relative values more convenient for expert than absolute ones [Saaty, 1996]. These values are used under paired comparison matrix forming as preference relation. Thus paired comparison matrix (PCM) contains an expert preference structure. We will use scale points of attributes as an PCM comparative alternatives. In the paper we will consider the problem of utility functions construction on paired comparison matrix base.

Preference representation on scale points

Let Z be scale points set. Then preference relation R on a set Z is subset on the product $Z\times Z$: $R\subset Z\times Z$. When cardinality of Z is small, the preference relation may be conveniently represented by the $n\times n$ matrix A. Its element a_{ij}, $\forall i, j\in\{1,\ldots,n\}$, is interpreted as the preference degree of the scale points z_i over z_j.

* The work had been fulfilled under financial support of Russian Fundamental Research Fund (project № 10-01-00439)

We will consider three kinds of preference relation: binary relation with $a_{ij}\in\{0, 1\}$, probabilities relation with $a_{ij}\in[0, 1]$, multiplicative or ratio relation with $a_{ij}\in[1/N, N]$, where $a_{ij}=N$ denotes that attribute useful in point z_i is N times as good as in point z_j.

These relations are represented by reciprocal matrixes. For binary and probabilities matrixes a_{ij}, $\forall i$, $j\in\{1,\ldots,n\}$, are calculated as $a_{ji}=1 - a_{ij}$. For multiplicative preference matrix $a_{ji} = 1/a_{ij}$. To construct reciprocal matrix it is need to make $n(n-1)/2$ comparisons between alternatives.

Beside reciprocal matrixes we will use non-reciprocal matrixes in which $a_{ji}\neq f(a_{ij})$. We will name the kind of preference represented by a such matrix as benefit / losses. For example, if footballs team *A* have wined team *B* with 3:1 score, matrix element a_{ij}=3 is interpreted as the benefit of team *A* and a_{ji}=1 is interpreted as the loss of that team. To construct non-reciprocal matrix it is need to make $n^2 - n$ comparisons between alternatives.

Hence there are some ways to construct consistent matrix from the set of n or $n - 1$ comparison. To construct non-reciprocal matrix it is enough to assess one from its rows or columns. Using known values of the first column a rows of the matrix are formed. All cells of the row accept the value from the first cell. If a values of the first column are rising from up to down, then $a_{ji} > a_{ij}$, $i\neq j$, $\forall i$, $j\in\{1,\ldots,n\}$. Another words all elements of the bottom triangle sub matrix are bigger then a corresponding elements of the upper triangle sub matrix. Analogous the matrix is constructed on the base of the known first row. That matrix contains the opposite preferences, because all elements of the upper triangle sub matrix are bigger then a corresponding elements of the bottom triangle sub matrix.

Another way of a matrix construction is to assess the cells of a matrix corresponding Hamilton path on their graph. The procedure demands only $n - 1$ values of preference relation. That values are entered into the cells of matrix parallel their main diagonal. On the next step a values are calculated for remaining cells of a matrix. There are developed a methods of single-digit finding of values for remaining cells of a matrix [Alonso S. et al.]. In paper [Kiselev 1, 2011] the task is solved as optimization one with new consistency criterion.

When all cells of paired comparison matrix have been assessed we can calculate the dominance degree between scale points. A paired comparison matrix only captures the dominance of one scale point over each other points in one step. The dominance is accumulated by raising the matrix to the next power beginning the first one. Tomas Saaty had proved how to obtain a relative scale among n alternatives from their paired comparison matrix. The relative dominance of an alternative is given by the solution of the eigenvalue problem $Aw=\lambda_{max}w$. Normalized eigenvector corresponding matrix eigenvalue $=\lambda_{max}$ represents finite dominance vector $w=(w_1,\ldots,w_j,\ldots,w_n)$ of the alternatives, where

$$\sum_{j=1}^{n} w_j = 1 \qquad (1)$$

In fact dominance a vector w represents a discrete priority function determined on the scale points. To receive a discrete utility function u from a priority function w the last one transforms by $(w_i - w_{min})/(w_{i,max} - w_{min})$. Values of utility function are belonged to interval [0, 1]. It should be noted that the linear dependence exists between utility function (UF) and priority function (PF). So it is enough to investigate the only priority function properties.

Linear utility functions construction

To create linear priority function it is necessary to maintenance even change of dominance of the *i*-th scale point over *j*-th point, $\forall i$, $j\in\{1,\ldots,n\}$, $i\neq j$. Dominance change magnitude depend on a kinds of preference and a matrix content. Even change of dominance is satisfied by a binary matrix which consist of the triangle sub matrix with cells a_{ji}=1 and another triangle sub matrix with cells a_{ij}=0. In such matrix the dominance difference between neighboring points equal 1. In the Table 1 the example of matrix 6 × 6 is shown.

The binary matrix is placed on the left side of the table 1. Its diagonal elements a_{ii}=1, $\forall i, j \in 1,\ldots,n\}$ to receive the smallest priority $w_{i,\min}$>0. The numbers in the column "Score" are the sums of ones in the corresponding matrix rows. Priority function values are calculated by score numbers normalization. In the column "Useful" a values calculated on base of the priority function are placed.

Table 1

Scale points	1	2	3	4	5	6	Score	Priority	Useful
1	1	0	0	0	0	0	1	0,0476	0,1665
2	1	1	0	0	0	0	2	0,0952	0,3331
3	1	1	1	0	0	0	3	0,1428	0,4995
4	1	1	1	1	0	0	4	0,1904	0,6660
5	1	1	1	1	1	0	5	0,2380	0,8325
6	1	1	1	1	1	1	6	0,2859	1,0000

The linear graphic of the utility function is shown in Fig. 1.

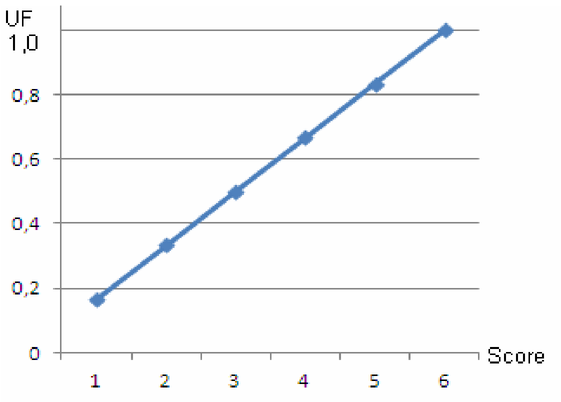

Fig. 1.

The discrete function points are connected to see the function form. The utility function with descending form is created on base of binary matrix which consist of the upper triangle sub matrix with cells a_{ji}=1 and bottom triangle sub matrix with cells a_{ij}=0.

A linear utility function can be created on the base matrix "benefit / losses" too. An example of such matrix is shown in Table 2.

Table 2.

Scale points	1	2	3	4	5	6	Score	Priority	Useful
1	1	1	1	1	1	1	6	0,05405	0,19353
2	2	1	2	2	2	2	11	0,09910	0,35484
3	3	3	1	3	3	3	16	0,14414	0,51611
4	4	4	4	1	4	4	21	0,18919	0,67742
5	5	5	5	5	1	5	26	0,23423	0,83869
6	6	6	6	6	6	1	31	0,27928	1,00000

The cells of each matrix row beside diagonal cells have the same values equal to cell values of the first column. The column sell values (1, 2, 3, 4, 5, 6) are obtained with a such generating function as arithmetic progression with step 1.

Non-linear utility functions construction

To create non-linear priority function it is necessary to maintenance variable change of dominance of the *i*-th scale point over *j*-th point, $\forall i, j \in \{1,\ldots,n\}$, $i \neq j$. This objective can be achieved by three ways: corresponding generating function choice, transforming one preference kind to another, change of parameter of priority function calculation on base PCM. This parameter is power of raising *k* the matrix A.

The simplest monotonic generating functions are geometric progression with constant step and Fibonacci function. An example of geometric progression is 2^i function, $i \in \{1,\ldots,n\}$. The function generates number consequence: 1, 2, 4, 8, 16,… The Fibonacci function generates number consequence: 1, 1, 2, 3, 5, 8,… The difference change between neighboring number consequence magnitudes determines the velocity of increasing or decreasing function. An example of non-monotonic generating function is Newton binomial one. A generating function can be applied for assessment of the first column or row the matrix and Hamilton path cells of corresponding graph.

In Fig. 2 the example is shown of non-linear priority function construction by transforming preference "benefit / losses" preference kind a_{ij}^{bl} (see matrix in Table 2) to probabilities preference kind a_{ij}^{pr} according formula:

$$a_{ij}^{pr} = \frac{a_{ij}^{bl}}{a_{ij}^{bl} + a_{ji}^{bl}} \quad (2)$$

Two curved shown in Fig. 2 characterize non-inclination decision maker to risk.

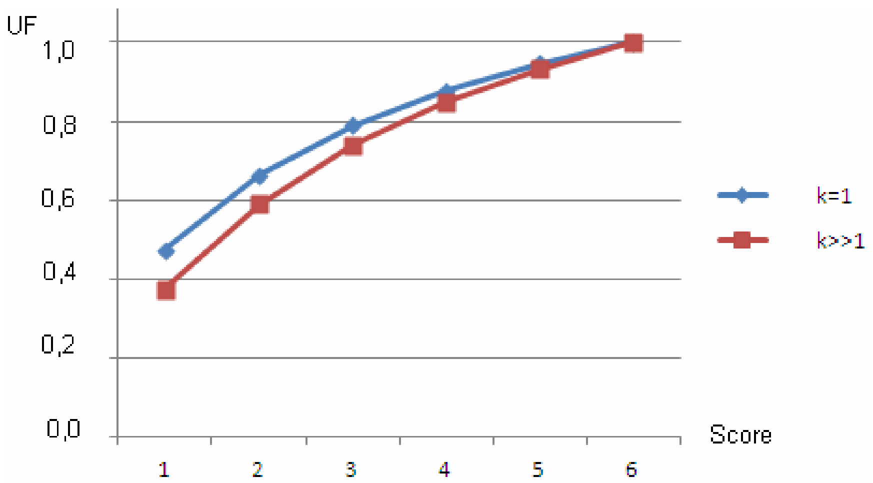

Fig. 2.

The upper utility function calculated by raising the matrix to first power (*k*=1) and bottom utility function calculated on base eigenvector of matrix by raising the matrix to power *k*>>1. Thus the example demonstrates both way of maintenance variable change of dominance of the *i*-th scale point over *j*-th point, transforming one preference kind to another and change of parameters of priority function calculation on base PCM.

Utility functions construction under known PCM

In multi criteria tasks beside quantitative criteria are often applied qualitative criteria too. For example, quality work can be characterized by manufacturing firm. To calculate value of multi attributes useful function it is necessary to transform firm names into numerical estimations. It can be made with expert help. Hence more objective estimations can be received if it is known results of firm interaction. Let firm interaction be meant patents trading and each firm is interested to sell more patents then to buy them. That firm interaction is represented by "benefit / losses" matrix shown in Table 3.

Under PCM the graphic of the utility function (UF) calculated on the base its eigenvector (matrix power k>>1) is shown in Fig. 3. To facilitate analyses the function values are replaced under corresponding matrix columns (firm names). The matrix shown in Fig. 3 has a bad consistency. It is confirmed by circles passing through corresponding graph vertexes. Number of circles passing through each vertex is shown in table 3 last row. The total number of circles into graph is equal 12 and maximum number is equal 20.

Table 3

Firm name	1	2	3	4	5	6	Score	UF, k=1	UF, k>>1
1	1	3	2	4	1	2	13	1,0000	0,9686
2	1	1	2	1	2	1	8	0,6154	0,6011
3	4	2	1	3	1	2	13	1,0000	1,0000
4	3	1	4	1	2	1	12	0,9231	0,9480
5	1	3	2	2	1	1	10	0,7692	0,7266
6	3	1	1	3	2	1	11	0,8462	0,8508
Cycles	5	7	7	6	5	6			

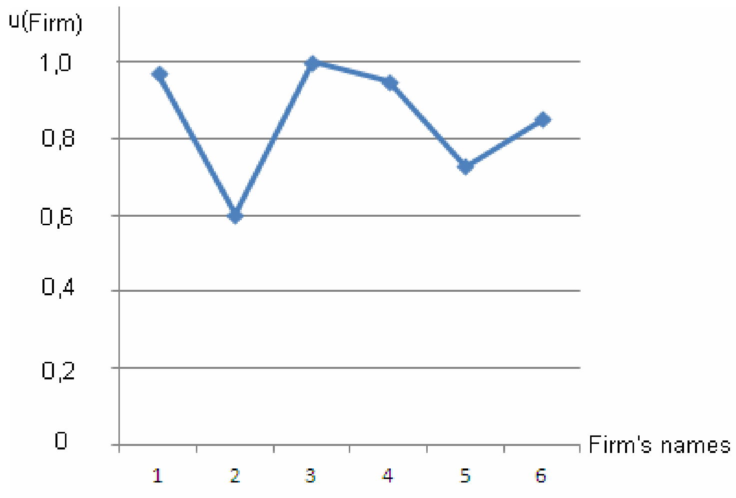

Fig. 3.

A biggest percent of circles into the graph indicates on a bad consistency of PCM. Hence consistency concept don't applied to matrixes presented a competition results. To evaluate competitor's aggression level Igor Kiselev had proposed total preference factor [Kiselev 2, 2011].

The priority functions perturbation under increasing the matrix power is well seen in Fig. 4. The matrix power corresponds abscises axis represented in logarithmic scale. The vertical line in the Fig. 4 indicates matrix power *k*=1 or ln *k* = 0. On the left side from point ln *k* = 0 priority functions of alternatives are aspirated to 1/*N* that corresponds to matrix power *k* = 0. On the right side from point ln *k* = 0 priority functions of alternatives are aspirated to the eigenvector. The eigenvector values are marked on right vertical line. In that matrix power point the perturbation process is over. It was reason why the UF with *k*>>1 had been choose in Fig. 3.

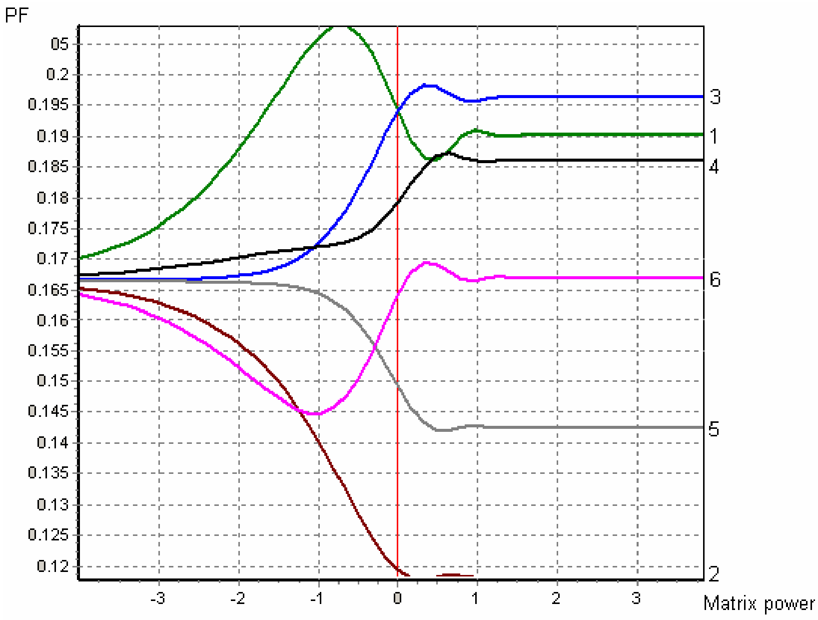

Fig. 4.

Conclusion

In addition to well-known methods of utility functions construction the new method is proposed based on paired comparison matrix using. We use scale points of attributes as a PCM comparative entities. The main problem of such approach is to assess the matrix content. Two ways of a problem solving are applied. One from them is expert method and another way is to tournament matrix using. To facilitate expert assessment of matrix content the shorten way is proposed. That way is based on generating function using. With a generating function help only *n* or *n* – 1 sells is assessed. The remaining cells are assessed automatically under consistency factor ensuring. The decision maker must only choice preference kind of matrix and generating function type.

Tournament matrixes are used when results of active entities interaction are known. They are applied for transforming of nominal values to numerical ones, for instance to assess firms importance. Such assessments permit us to use qualities' attributes under multi-attribute values computing.

The convention and effectively of proposed method was confirmed by numerous experiments on PCM applying for utility functions construction. The all experiments had been fulfilled on the choice and ranking system "SVIR-R". The system had been elaborated in St. Petersburg State Transport University under author direction [www.mcd-svir.ru].

Bibliography

[Saaty, 1996] Saaty T.L. The Analytic Network Process: Decision Making With Dependence and Feedback. RWS Publications, 1996. P. 370.

[Alonso S. et al.] A consistency based procedure to estimate missing pair wise preference values // International Journal of Intelligent Systems. 2009. Vol. 23, № 2. P. 155–175.

[Kiselev 1, 2011] Index of quantitative preference consistency in pair wise comparison matrix // Bulletin of the Tomsk Polytechnic University. Не все данные статьи!

[Kiselev 2, 2011] название автореферата

Authors' Information

Stanislav Mikoni – professor of St. Petersburg State Transport University, St. Petersburg 190031, Russia; e-mail: svm@sm4265.spb.edu

Major Fields of Scientific Research: System analyses, Multi-attribute choice, intellect technologies.